Adaptive Bone Remodeling Incorporating Cosserat Theory and Anisotropy

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Summary

This study offers a novel perspective on bone remodeling, contrasting classical elasticity theory with the Cosserat theory of elasticity. By treating a bone segment as an anisotropic material, this research incorporates Cosserat theory into the adaptive elasticity framework to explore bone's mechanical characteristics. Focused on the proximal part of the human femur, the model was analyzed using COMSOL Multiphysics finite element software. Results indicate that Cosserat theory, within the context of adaptive bone remodeling, provides a more accurate representation of bone behavior during the remodeling process.

Introduction

Bone's mechanical properties are crucial for supporting movement and protecting organs, requiring accurate modeling under mechanical loads. Traditional models based on classical elasticity fall short in capturing complex microstructures. This study applies Cosserat elasticity, incorporating rotational degrees of freedom and couple stresses, to model adaptive bone remodeling. By treating the proximal femur as an orthotropic material, the model improves predictions of displacement, density distribution, and stress localization, addressing classical elasticity's limitations.

Methods

The impact of Cosserat elasticity on adaptive bone remodeling was investigated using finite element (FE) simulations. A 3D proximal femur model, derived from CT-scan data, was meshed and analyzed using COMSOL Multiphysics. Cube specimens ($1mm^3$) were extracted from cancellous and cortical bone regions for detailed analysis (Figure 1). The constitutive equations of Cosserat elasticity, incorporating stress and couple stress tensors to account for rotational degrees of freedom, were formulated and implemented, as presented in Equations (1) and (2). Adaptive bone remodeling, introduced by Huiskes et al. [1] and expanded by Li et al. [2], was modeled using Equations (3) – (5).

$$\boldsymbol{\sigma} = \lambda tr[\boldsymbol{\varepsilon}]\boldsymbol{I} + 2\mu(sym\boldsymbol{\varepsilon}) + 2\mu_c(skew\boldsymbol{\varepsilon}) \qquad (1)$$

$$\boldsymbol{m} = \alpha tr[\nabla\boldsymbol{\varphi}]\boldsymbol{I} + \beta\nabla\boldsymbol{\varphi}^T + \gamma\nabla\boldsymbol{\varphi} \qquad (2)$$

where $\alpha, \beta, \gamma, \mu_c$, and l_c are Cosserat parameters, $\boldsymbol{\sigma}$ and $\boldsymbol{m}$ represent stress and couple stress tensors respectively, $\boldsymbol{\varphi}$ denotes the rotation vector, λ and μ are the classical Lamé moduli and $\boldsymbol{\varepsilon}$ stands for strain tensor.

$$\frac{\partial\rho}{\partial t} = B\left(\frac{U}{\rho} - (1-\delta)\gamma\right) \quad \text{if } \frac{U}{\rho} < (1-\delta)\gamma \qquad (3)$$

$$\frac{\partial\rho}{\partial t} = 0 \qquad \text{if } (1-\delta)\gamma \le \frac{U}{\rho} \le (1+\delta)\gamma \qquad (4)$$

$$\frac{\partial\rho}{\partial t} = B\left(\frac{U}{\rho} - (1+\delta)\gamma\right) - D\left(\frac{U}{\rho} - (1+\delta)\gamma\right)^2 \quad \text{if } \frac{U}{\rho} > (1+\delta)\gamma \qquad (5)$$

Results and Discussion

The Cosserat model showed reduced displacements with increasing internal length scale, as shown in Figure 1, with a maximum reduction of 36.33% compared to classical elasticity at a length scale of 10. Stress concentrations and strain energy density also decreased, while the added rotational energy improved load-bearing capacity, resulting in slower remodeling rates and lower bone density. These findings align with clinical data, highlighting the superior accuracy of Cosserat elasticity in modeling bone behavior.

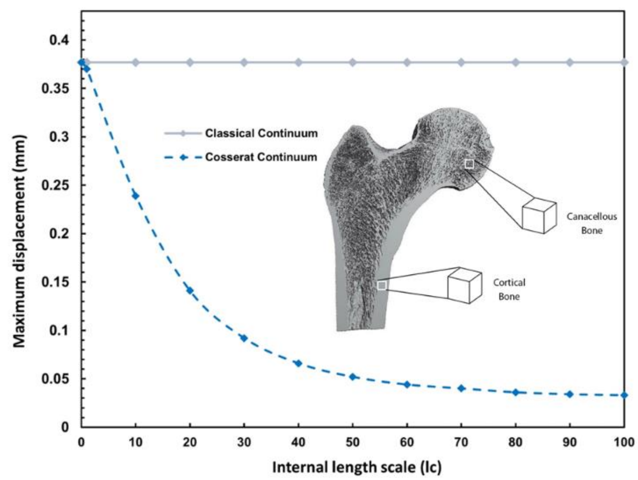

Figure 1: The maximum displacement over 48 days for both approaches across different internal length scales.

By incorporating size effects and microstructural behavior, this approach provides more accurate and realistic results than classical elasticity, advancing biomechanics and clinical solutions.

Conclusions

The study concludes that applying Cosserat elasticity in adaptive bone remodeling provides more accurate results than classical elasticity, particularly for materials with microstructures like bone.

References

[1] Huiskes R, et al. (1987) *Journal of biomechanics* **20.11-12**: 1135-1150.
[2] Li J, et al. (2007) *Dental materials* **23.9**: 1073-1078.