

# A modified theory of bending tests for bone with microstructure consideration

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## Summary

This study considers the microstructure and the real cross section of bone in the development of the theory of bending tests. A viscoelastic tension-shear chain model is adopted as the material-point model and embedded into a beam theory. Then, the real cross section of bones is considered and then the load-deflection formulations of the three-point and the four-point bending tests are derived. Experimental validation was conducted using three-point bending tests on rat femurs, revealing strong agreement between theoretical predictions and observed data. These findings highlight the importance of considering microstructural variations and material properties in understanding bone mechanics, with implications for aging, disease progression, and fracture risk assessment.

## Introduction

The three-point or the four-point bending testing is an important technique of experiments for the exploration on the mechanical property of materials and this technique bases on the assumption including Euler-Bernoulli beam theory, linearly elastic constitution, and prismatic segment. However, the natural bone cannot fit the assumption completely and the Euler-Bernoulli beam theory has a bias against the experimental data [1], therefore a modification of theory for the bending testing of bone is necessary if we need to explore the real property of bone. Furthermore, bones, as complex hierarchical composites, consist of staggered microstructure of mineral platelets and collagen matrix [2]. Therefore, such microstructure of bones should be considered if the modified technique is developed.

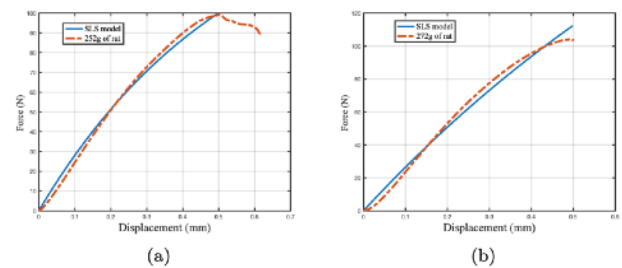
## Methods

The conventional tension-shear chain (TSC) model [3] was extended to take the viscoelastic behavior of collagen into account and a viscoelastic TSC model was developed. The standard linear solid model of viscoelasticity was be the model of collagen and the elastic model for mineral. This modified TSC model enables us to explore the rate-dependent properties of bones. Then, the viscoelastic TSC model is designated as the material-point model and corresponding beam theory is established. Furthermore, the varying cross sections of bones are adopted to derive the load-deflection formulation.

## Results and Discussion

The modified theory of bending tests was validated by the three-point bending tests on rat femurs, considering variations in cross-sectional geometry of the femurs captured via micro-computed tomography (micro-CT). The result of the three-point bending tests shows a strong correlation between theoretical predictions and experimental data. Specifically, the load-deflection curves derived from the modified beam theory closely matched the experimental

results, highlighting the accuracy of the proposed model. These findings confirm the importance of incorporating viscoelastic properties as well as microstructural variations into bone modeling. They also suggest that the developed model has potential applications in predicting fracture risk and understanding how bone properties change with age or disease. The alignment between theoretical and experimental results underscores the robustness of the proposed approach and its value in advancing the study of bone mechanics.



**Figure 1:** Comparison between experimental results and theoretical model predictions for mouse femurs of different weight under three-point bending tests. (a) 252 g, (b) 272 g.

## Conclusions

This research successfully integrates a viscoelastic constitutive law into bone micromechanics, validated through experimental bending tests. By accounting for microstructural staggering patterns and rate-dependent behavior, the modified TSC model provides a robust framework for studying bone mechanics. Future work will focus on applying these findings to clinical contexts, such as predicting fracture risk and understanding aging-related bone changes.

## References

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