Recommended effect size interpretations for one-dimensional functional data analysis

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Summary

We present effect size (Cohen's d-value) interpretations for functional data that are probabilistically equivalent to conventional interpretation guidelines for scalar data. Results suggest that traditionally "Large" effects occur with a high probability in functional data, and ought to be regarded as "Small" or "Very small" for probabilistic consistency.

Introduction

Cohen's d-values are often interpreted according to the guidelines of Sawilowsky (2009) for simple scalar quantities (Table 1) [1]. A major limitation of these guidelines is that random fluctuations in smooth functional data (e.g. joint angle trajectories) generally yield a given effect size with much greater probability than scalar data (Fig.1) [2]. This probability is directly related to smoothness, measured as the parameter FWHM or "W" (approximately the inverse of the average first derivative).

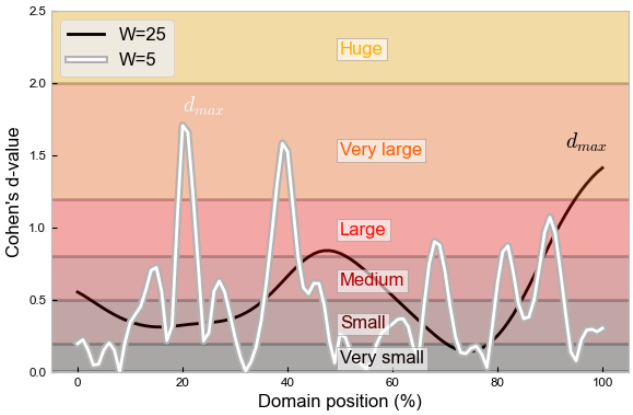

Figure 1: Example functional effect sizes (Cohen's d) when there is zero true effect. Each depicted d-value function was produced by a single random sample (two groups each with N=10) with a true population effect of zero.

Methods

We first calculated the p-values associated with the standard d-value interpretations, assuming a two-sample test with group sizes of N=10 (Table 1). We then used random field theory (RFT) [3] to calculate probabilistically equivalent functional d-values for a range of functional smoothness values (Table 1, right two columns).

Table 1: Proposed effect size interpretations for 1D functional data. P-values are probabilities associated with Sawilowsky (2009)'s d-values for a two-sample test with group sizes of N=10. The final two columns contain functional d-max values with the same probabilities for smoothness values of W=25 and W=5.

Label	Cohen's d	p-value	Functional d-max	
			(W=25)	(W=5)
Very small	0.01	0.491	0.54	1.00
Small	0.2	0.330	0.72	1.14
Medium	0.5	0.139	1.00	1.38
Large	0.8	0.045	1.29	1.64
Very large	1.2	0.008	1.70	2.05
Huge	2.0	0.001	2.59	2.96

Results and Discussion

RFT shows that any given effect size occurs randomly with much greater probability in functional data. Probabilistically equivalent interpretations to Sawilowsky's guidelines suggest that effects traditionally regarded as "Large" should be regarded as "Small" or even "Very small", depending on functional smoothness. Functional effect sizes therefore ought to be interpreted more cautiously than simple scalar effect sizes. We intend to make more comprehensive and easy-to-follow guidelines publicly available soon.

References

[1] Sawilowsky SS (2009). *JMASM* **8**: 597–599.
[2] Pataky TC et al. (2016). *J.Biomech* **49**: 1468–1476.
[3] Worsley KJ et al. (2004). *Neuroimage* **24**: S189–S195.